Note on Binaural Masking-Level Differences as a Function of the Interaural Correlation of the Masking Noise

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(Received 21 May 1964)

A quantitative "black-box" model is constructed for interpreting data that describe the dependence of binaural masked thresholds on the proportion of common noise in the masking signals. This model is a natural adjunct to the equalization-and-cancellation model, discussed in previous papers.

IN the writer's previous theoretical work on binaural masking-level differences,[1–4] it has always been assumed that the interaural relations of the two masking signals are deterministic ones.[5] In the present note, a model is constructed for data in which the interaural relations are statistical. The data considered are those of Robinson and Jeffress, describing the variation in the binaural masked threshold as a function of the crosscorrelation coefficient of the masking components.[6] The model is a natural adjunct to the equalization-and-cancellation model and it is assumed that the reader is familiar with the writer's main paper on this topic.[3]

In the Robinson–Jeffress experiment, the two masking components were synthesized from three separate noise sources. One source was fed to one ear, one to the other ear, and one to both ears. The spectra of these three sources were all the same and covered the frequency region 100–3000 cps. The levels of the first two sources were always equal, and the levels of the two channels from the third source were always equal. The level of the total masking signal (the same for each ear) was maintained at 50 dB *re* 0.0002 μbar, and the crosscorrelation coefficient of the masking signals presented to the two ears was varied by controlling the proportion of noise contributed by the source that was common to both ears. The target signal consisted of a 500-cps tone of 150-msec duration, having a rise and decay time of 25 msec, and was presented with equal amplitude to both ears. In addition to varying the proportion of common noise in the masking signals, the following four interaural phase configurations were considered.

N0S0	common noise and signal both in phase at the two ears
NπSπ	common noise and signal both reversed in phase at one ear
N0Sπ	common noise in phase, signal reversed in phase at one ear
NπS0	common noise reversed in phase at one ear, signal in phase

Also, two methods of threshold measurement were employed, one based on the "constant" method and one based on the "two-alternative, forced-choice" method.

The total stimuli $y_1(t)$ and $y_2(t)$ presented to ears 1 and 2 in this experiment can be written

$$\begin{aligned} y_1(t) &= us(t)+n_1(t), \\ y_2(t) &= s(t)+n_2(t), \end{aligned} \tag{1}$$

where $s(t)$ is the target signal, $n_j(t)$ is the masking signal to ear j $(j=1,2)$, and $u=\pm 1$. The masking signals $n_j(t)$ can be written

$$\begin{aligned} n_1(t) &= vam_c(t)+bm_1(t), \\ n_2(t) &= am_c(t)+bm_2(t), \end{aligned} \tag{2}$$

[1] N. I. Durlach, "Note on the Equalization and Cancellation Theory of Binaural Masking-Level Differences," J. Acoust. Soc. Am. 32, 1075–1076 (1960).

[2] N. I. Durlach, "Note on the Creation of Pitch through Binaural Interaction," J. Acoust. Soc. Am. 34, 1096–1099 (1962).

[3] N. I. Durlach, "Equalization and Cancellation Theory of Binaural Masking-Level Differences," J. Acoust. Soc. Am. 35, 1206–1218 (1963).

[4] N. I. Durlach, "Note on Binaural Masking-Level Differences at High Frequencies," J. Acoust. Soc. Am. 36, 576–581 (1964).

[5] In the data toward which the previous theoretical work has been oriented, the masking signals have always been identical except for a fixed shift in amplitude, time of arrival, or phase.

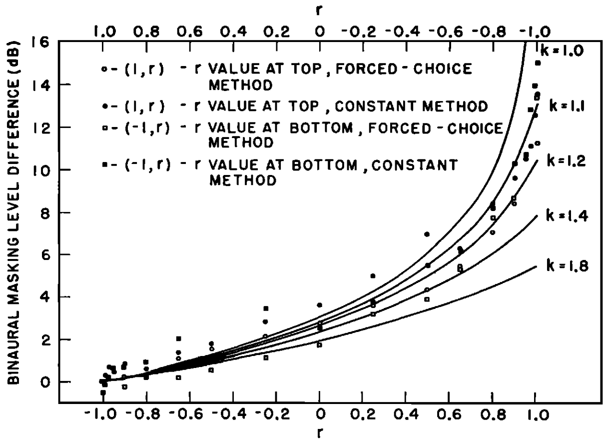

FIG. 1. Binaural masking-level difference as a function of the interaural correlation coefficient of the masking noise. Data obtained from Robinson and Jeffress.

where $m_c(t)$ is the noise waveform common to both ears, $m_j(t)$ the noise waveform fed to ear j ($j=1,2$), a, b the amplitude factors, and $v=\pm 1$.

Assume now that the noise waveforms $m_c(t)$, $m_1(t)$, and $m_2(t)$ are normalized to have unit power. The total power in $n_j(t)$ is then given by a^2+b^2 and the proportion of power that is due to the common noise is given by $a^2/(a^2+b^2)$. The correlation coefficient r of the masking signals $n_j(t)$ is defined by the equation

$$r=\langle n_1(t)n_2(t)\rangle/\langle n_1^2(t)\rangle^{\frac{1}{2}}\langle n_2^2(t)\rangle^{\frac{1}{2}}, \tag{3}$$

where the angular brackets denote the ensemble average. Making use of Eqs. (2), one obtains

$$r=va^2/(a^2+b^2). \tag{4}$$

The magnitude of r depends upon the proportion of common noise $a^2/(a^2+b^2)$, and the sign of r depends upon whether the common noise was presented in phase ($v=1$) or out of phase ($v=-1$). With this notation, each of the stimuli used in the experiment can be specified by choosing the values of u and r in the pair (u,r) as follows:

N0S0 $(u=1,\ 0<r\leqslant 1)$,

NπSπ $(u=-1,\ -1\leqslant r<0)$,

N0Sπ $(u=-1,\ 0<r\leqslant 1)$,

NπS0 $(u=1,\ -1\leqslant r<0)$.

The condition $r=0$ is equivalent to no common noise at all.

A summary of the experimental data obtained by Robinson and Jeffress is shown in Fig. 1. The abscissa at the bottom of the graph gives the value of r for the configuration $(-1,r)$. The abscissa at the top of the graph gives the value of r for the configuration $(1,r)$. The ordinate presents the masking-level difference between the given condition and the "standard" condition (1,1). The curves in Fig. 1 are theoretical and are explained below. For further details on the experimental data, the reader is referred to the paper by Robinson and Jeffress.[6]

Assume now that (a) the listener's detection performance is determined by the signal-to-noise ratio at the output of the binaural interaction, and (b) the auditory system's scheme for improving this ratio (over the monaural signal-to-noise ratio) is to add the two stimuli in the case $(1,r)$ and to subtract the two stimuli in the case $(-1,r)$. Assume, furthermore, that (c) there exists a bandpass filter in the ear (the "critical band") that filters out the noise outside a band around the frequency of the target signal, and (d) the addition and subtraction operations are corrupted by random amplitude and time errors. [The assumption that the auditory system responds to the stimulus $(1,r)$ by adding is replaced below by the assumption that it responds by inserting an interaural delay of one-half cycle of the signal tone and then subtracting. The reason for considering the addition postulate first is that it leads to a model that is simpler. The reason for considering the delay-subtraction postulate is explained below.]

Denoting the result of passing a signal through the filter by a prime and denoting the amplitude and time errors by ϵ_j and δ_j, one can express the output of the binaural interaction $Y(t)$ in the form

$$\begin{aligned} Y(t)&=(1-\epsilon_2)y_2'(t-\delta_2)+u(1-\epsilon_1)y_1'(t-\delta_1)\\ &=\mathrm{S}(t)+\mathrm{N}(t), \end{aligned} \tag{5}$$

where

$\mathrm{S}(t)$, signal component after binaural interaction,
$=(1-\epsilon_2)s'(t-\delta_2)+(1-\epsilon_1)s'(t-\delta_1)$,

$\mathrm{N}(t)$, noise component after binaural interaction,
$=(1-\epsilon_2)[am_c'(t-\delta_2)+bm_2'(t-\delta_2)]$
$+u(1-\epsilon_1)[vam_c'(t-\delta_1)+bm_1'(t-\delta_1)]$.

Inasmuch as the bandwidth of the target signal is substantially less than the critical bandwidth, one has $s'(t)=s(t)$. Using the same letter to denote the Fourier transform of a function as was used to denote the time function[7] (and letting ω denote angular frequency), one can rewrite Eqs. 1, 2, and 5 as

$$y_1(\omega)=us(\omega)+n_1(\omega),$$
$$y_2(\omega)=s(\omega)+n_2(\omega),$$
$$n_1(\omega)=vam_c(\omega)+bm_1(\omega),$$
$$n_2(\omega)=am_c(\omega)+bm_2(\omega),$$
$$Y(\omega)=\mathrm{S}(\omega)+\mathrm{N}(\omega),$$

[6] D. E. Robinson and L. A. Jeffress, "Effect of Varying the Interaural Noise Correlation on the Detectability of Tonal Signals," J. Acoust. Soc. Am. 35, 1947–1952 (1963).

[7] It is assumed here that all stimulus functions are defined to be zero outside some finite time interval so that the Fourier transforms of these functions converge.

$$S(\omega)=s(\omega)[(1-\epsilon_2)\exp(-i\omega\delta_2)+(1-\epsilon_1)\exp(-i\omega\delta_1)], \quad (6)$$

$$N(\omega)=am_c'(\omega)[(1-\epsilon_2)\exp(-i\omega\delta_2)+uv(1-\epsilon_1)\exp(-i\omega\delta_1)]+b[(1-\epsilon_2)m_2'(\omega)\exp(-i\omega\delta_2)+u(1-\epsilon_1)m_1'(\omega)\exp(-i\omega\delta_1)].$$

Assume now that (e) the appropriate signal-to-noise ratio at the output of the binaural interaction is obtained by averaging over the errors ϵ_j and δ_j before dividing the signal component by the noise component. Letting AV denote the average over the errors (and recalling that angular brackets denote the average over the noise ensemble), one can express the change in signal-to-noise ratio between the monaural and binaural cases by the factor

$$f=\frac{\int \text{AV}|S(\omega)|^2d\omega/\int \text{AV}\langle|N(\omega)|^2\rangle d\omega}{\int |s(\omega)|^2d\omega/\int \langle|n'(\omega)|^2\rangle d\omega}. \quad (7)$$

In this equation, the subscript on $n'(\omega)$ has been omitted because $\langle|n_1'(\omega)|^2\rangle=\langle|n_2'(\omega)|^2\rangle$.

Assume, finally, that (f) ϵ_1, ϵ_2, δ_1, δ_2 are statistically independent random variables of mean zero and variances $\sigma_\epsilon^2=\sigma_{\epsilon_1}^2=\sigma_{\epsilon_2}^2$ and $\sigma_\delta^2=\sigma_{\delta_1}^2=\sigma_{\delta_2}^2$; (g) δ_1 and δ_2 are Gaussian; (h) the change in $\exp(-\omega^2\sigma_\delta^2)$ across the critical bandwidth is small. One can then show that, approximately,

$$f=\frac{1+\sigma_\epsilon^2+\exp(-\omega_0^2\sigma_\delta^2)}{1+\sigma_\epsilon^2+[a^2/(a^2+b^2)]uv\exp(-\omega_0^2\sigma_\delta^2)}, \quad (8)$$

where ω_0 denotes the frequency of the target signal. Making use of Eq. (4), one can rewrite Eq. (8) as

$$f=(k+1)/(k+ur), \quad (9)$$

where $k=(1+\sigma_\epsilon^2)/\exp(-\omega_0^2\sigma_\delta^2)$. For the standard stimulus (1,1), one has $f=1$. In other words, the threshold for this stimulus is the same as for the monaural stimulus.[8] Thus, according to the model, the quantity $10\log f$ constitutes the theoretical counterpart of the data in Fig. 1.

In the event that the system performs with perfect precision (i.e., $\sigma_\epsilon^2=\sigma_\delta^2=0$), one obtains $k=1$ and $f=2/(1+ur)$. If, also, there is no common noise (i.e., $r=0$), then $f=2$ and the improvement over the standard condition is 3 dB. If, instead, $r=u$, then $f=1$ and there is no improvement. If, on the other hand, $r=-u$, then $f=\infty$ and the improvement is infinite. Returning to the general case given by Eq. (9), one notes that for $u=-u$ Eq. (9) reduces to the same equation as that which was used previously[9] by the writer for the case $(-1, 1)$.

The curves in Fig. 1 are obtained from Eq. (9) by inserting the values 1.0, 1.1, 1.2, 1.4, and 1.8 for k. If the values $\sigma_\epsilon=0.25$ and $\sigma_\delta=105$ μsec (which were frequently used in the writer's previous work[3]) are employed, one obtains $k=1.2$. The fact that the theoretical curve for this value of k falls below the data at $(-1, 1)$ is merely a reflection of the fact that the curve selected for determination of the values σ_ϵ and σ_δ in the writer's previous work fell below the corresponding data.[10] The crucial test for evaluating the consistency of Eq. (9) with the data consists in comparing the data with the curve whose k value is chosen to fit the data at $(-1, 1)$ or $(1, -1)$, i.e., in comparing the shapes of the theoretical and empirical curves. Although no statistical analysis has been made, crude visual observation indicates that the fit is not unreasonable.[11]

Those readers who are familiar with the writer's previous work on the equalization-and-cancellation model[1-3] will observe that the concepts, techniques, approximations, etc., used above are essentially the same as those used before for different stimuli. One point of difference, however, is the following. In the writer's previous work,[12] it was suggested that, when the stimulus was of the form $(1,-1)$, the auditory system responded not by adding but by inserting an interaural delay of one-half cycle of the signal tone and then subtracting. This hypothesis was invoked in order to explain the masking-level difference between the conditions $(1, -1)$ and $(-1, 1)$, a difference that becomes as much as 7 dB at 200 cps, without resorting to changes in the error factors σ_ϵ and σ_δ. The value of the masking-level difference was then interpreted as being determined by the width of the critical band and the width of the head. The hypothesis that the system delayed and subtracted, rather than simply added, was "justified" on the grounds that the latter operation, unlike the former, was not ordinarily required in a natural acoustical environment. In order to increase the consistency of the present model with the previous one [and to account for the binaural masking-level differences between the stimuli $(1, -1)$ and $(-1, 1)$], it will now be assumed that the auditory system responds to the stimulus $(1, r)$ by delaying one-half cycle and subtracting.

Letting β denote the width (in radians) of the critical band and γ the mean value of $\delta_1-\delta_2$ (no longer necessarily assumed to be zero), one can show[13] that the improvement in signal-to-noise ratio for the stimulus

[8] The theoretical prediction that the stimulus (1,1) leads to the same threshold as the monaural stimulus is well-supported by experiment. For some data comparing these two stimuli, see, for example, I. J. Hirsh and M. Burgeat, "Binaural Effects in Remote Masking," J. Acoust. Soc. Am. **30**, 827–832 (1958).

[9] See Ref. 3, Eq. (14).

[10] Compare the theoretical curve and the data points at 500 cps labeled JBD, BJT, and JBSW in Ref. 3, Fig. 2.

[11] In the writer's opinion, the main deficiency in the shape of the theoretical curves is that they do not rise quite steeply enough in the region where f is large.

[12] See Ref. 3, pp. 1216–1217.

[13] In deriving Eq. (10), the following assumptions were made: (a) the change in $\cos[\omega(\pi/\omega_0-\gamma)]\exp(-\omega^2\sigma_\delta^2)$ across the frequency span of $|s(\omega)|^2$ is small; (b) $\langle|m_c'(\omega)|^2\rangle$ is a rectangle centered on ω_0; (c) $\sin[\beta(\pi/\omega_0-\gamma+\delta_1-\delta_2)/2]/[\beta(\pi/\omega_0-\gamma+\delta_1-\delta_2)/2]\approx\sin[\beta(\pi/\omega_0-\gamma)/2]/[\beta(\pi/\omega_0-\gamma)/2]$. Use was also made of the equality $\int \text{AV}\langle|N(\omega)|^2\rangle d\omega=\text{AV}\int\langle|N(\omega)|^2\rangle d\omega$.

$(1, r)$ achieved with the delay-subtraction operation is given by

$$\tilde{f}=\frac{[k/\cos(\omega_0\gamma)]+1}{[k/\cos(\omega_0\gamma)]+r\sin q/q}, \quad (10)$$

where $q=\beta(\pi/\omega_0-\gamma)/2$. The terms $\sin q/q$ arises from the decorrelation in the common-noise component caused by the delay $\pi/\omega_0-\gamma$. The possibility of a nonzero value for γ is introduced in order to account for a possible head-width limitation.[14] If it is assumed that the system cannot effect a delay greater than the time width h of the head (i.e., greater than the delays normally required) and $\pi/\omega_0>h$, then γ is given by π/ω_0-h and q becomes $\beta h/2$. The change in the masked threshold resulting from the change in the binaural processing (from addition to delay subtraction) is given by the ratio $f/\tilde{f}$. Combining Eqs. (9) and (10), one obtains

$$f/\tilde{f}=\frac{(k+1)[k+r\cos(\omega_0\gamma)\sin q/q]}{[k+\cos(\omega_0\gamma)](k+r)}. \quad (11)$$

If one assumes that the system has no added difficulty in effecting delays greater than h (so that γ can be taken equal to zero), then Eq. (11) reduces to

$$f/\tilde{f}=\frac{k+r\sin(\beta\pi/2\omega_0)/(\beta\pi/2\omega_0)}{k+r}. \quad (12)$$

Since $\beta\leqslant 2\omega_0$ and $0\leqslant\sin(\beta\pi/2\omega_0)/(\beta\pi/2\omega_0)<1$, one has $f/\tilde{f}\geq 1$ if and only if $r\leqslant 0$. Choosing a critical bandwidth of $\beta/2\pi=100$ cps (Ref. 15), the factor $\sin(\beta\pi/2\omega_0)/(\beta\pi/2\omega_0)$ becomes 0.984. Clearly, unless one chooses k and r such that $k+r$ is very small, one has $f/\tilde{f}\approx 1$. For $k=1.1$ and $r=-1$, one obtains $f/\tilde{f}=1.16$ or 0.6 dB. For $k=1.05$ and $r=-1$ (leading to a value of f of 41 or 16 dB), one obtains $f/\tilde{f}=1.32$ or 1.2 dB. In general, for reasonable choices of β and k, it is apparent that if one discounts the head-width limitation the difference between the addition and the delay-subtraction operations is reasonably small at $\omega_0/2\pi=500$ cps. It should also be noted that, to the extent that the differences are not negligible, the delay-subtraction operation gives a better fit to the data.[16]

Returning now to Eq. (11) and the head-width-limitation factor γ, one observes that $f/\tilde{f}$ is most sensitive to γ in the neighborhood of $r=-1$. Assuming that γ is confined to the interval 0 msec$\leqslant\gamma\leqslant$0.4 msec (corresponding to a minimum maximum-achievable-delay time of 0.6 msec), one has $1\geq\cos(\omega_0\gamma)\geq 0.31$. For $r=1$, one obtains $f/\tilde{f}=[k+\cos(\omega_0\gamma)\sin q/q]/[k+\cos(\omega_0\gamma)]\approx 1$, independent of γ. For $r=0$, one obtains $f/\tilde{f}=(k+1)/[k+\cos(\omega_0\gamma)]$ and finds that $f/\tilde{f}$ is bounded between 1.00 and 1.52, the lower bound occurring when $\gamma=0$ and the upper when $\gamma=0.4$ msec and $k=1.00$. When $r=-1$, one obtains

$$f/\tilde{f}=\left[\frac{k+1}{k+\cos(\omega_0\gamma)}\right]\left[\frac{k-\cos(\omega_0\gamma)\sin q/q}{k-1}\right]. \quad (13)$$

(Note that the first factor on the right-hand side of this equation is simply $f/\tilde{f}$ evaluated at $r=0$.) It is clear that, for an appreciably γ, and for k reasonably close to unity, $f/\tilde{f}$ can become quite large. For example, for $k=1.1$, $\sin q/q=0.984$, and $\gamma=0.1$, 0.2, 0.3, and 0.4, one obtains $f/\tilde{f}=1.68$, 3.34, 6.49, and 11.87, respectively. In general, in order to fit the present data by using subtraction for the stimulus $(-1, r)$ and delay subtraction for the stimulus $(1, r)$, and to maintain a common k for both stimuli, γ must be restricted to relatively small values.[17]

A second point that requires consideration when comparing this model with the previous work is the following. In the previous work, the basic scheme of the auditory system was described as an attempt to eliminate the masking noise by equalizing the two noise components and then subtracting. If this processing were performed with complete precision, and the interaural relations of the target signal differed from those of the masking signal, the masking signal would be completely eliminated and there would be an infinite improvement in the signal-to-noise ratio. In the present model, the operations chosen for the auditory system do not equalize and cancel the two masking components and, is some cases, actually lead to an increase in the noise level.[18]

In general, it is clear that in order to make the present model consistent with the previous model, both models must be regarded as special cases of a more general model. Although this more general model has not been worked out in any detail, it appears that the basic

[14] For a comparable treatment of the head-width-limitation problem for stimuli in which the masking signals are identical except for an interaural amplitude ratio and an interaural time delay, see Ref. 3, Eq. (7).

[15] See, for example, D. D. Greenwood, "Auditory Masking and the Critical Band," J. Acoust. Soc. Am. **33**, 484–502 (1961); J. A. Swets, D. M. Green, and W. P. Tanner, Jr., "On the Width of Critical Bands," J. Acoust. Soc. Am. **34**, 108–113 (1962). Note, in particular, the result in the latter paper that, if the critical-band filter is assumed to be rectangular [as was assumed in the derivation of Eq. (10)], the width at 1000 cps is approximately 100 cps. (In general, it has been found that the width of the critical band is essentially constant between 500 and 1000 cps.)

[16] For both the "constant" and "forced-choice" methods, the stimulus $(1,-1)$ leads to a slightly smaller masking-level difference than the stimulus $(-1,1)$. If the subtraction operation is assumed for the stimulus $(-1,r)$ and the delay-subtraction operation is assumed for the stimulus $(1,r)$, this difference is described theoretically by the difference between f and $\tilde{f}$.

[17] Note that a similar result was obtained for the data in Ref. 3, Fig. 8. The best-fit theoretical curve for those data was obtained by assuming a $\beta/2\pi$ of 100 cps and an h of 0.9 msec (corresponding here to a γ of 0.1 msec). The discrepancy between this value of h on the one hand and the more reasonable value $h=0.6$ msec obtained from the data in Ref. 3, Fig. 9, on the other hand, is currently being explored experimentally.

[18] For example, in applying the subtraction operation to the stimulus $(-1,r)$, not only are the independent components $m_1(t)$ and $m_2(t)$ combined to raise the noise level resulting from these components by a factor of 2, but, when the common component $m_c(t)$ is presented out of phase (i.e., $r<0$), the noise level resulting from this component is raised by a factor of 4.

scheme of this model should involve the following postulates: (a) there exists a given repertoire of interaural transformations available to the auditory system; (b) the binaural processing consists of applying one of these transformations and then subtracting; (c) for a given stimulus, the auditory system chooses that transformation which maximizes the signal-to-noise ratio at the output of the subtractor. For all of the data considered to date, it has been sufficient to assume that the repertoire of interaural transformations consists of the single transformation "delay." To what extent this repertoire will have to be enlarged when further data are considered and to what extent it can be altered by learning processes, remain to be seen.

The fact that the variation of the binaural masked threshold with the interaural correlation coefficient evidenced in the Robinson–Jeffress experiment differs so drastically from that evidenced in the Jeffress–Blodgett–Deatherage experiment[19] (in which the correlation was varied by controlling the interaural delay of the noise) is no surprise according to this model. If the auditory system is capable of inserting its own delay, what counts is not the interaural correlation coefficient of the noise but the values of the interaural correlation function of the noise over the region of possible compensating delays. Unlike the case in which the decorrelation is achieved by adding independent noise sources, if the correlation coefficient of the noise stimulus is decreased by use of a delay, it can be increased again by inserting a delay of equal magnitude and opposite sign. Whether or not this "recorrelation" can be achieved by the auditory system will depend on whether or not the required compensating delay is in the auditory system's repertoire of interaural transformations (i.e., on the magnitude of the delay). According to recent work by Langford and Jeffress,[20] the dependence of the binaural masked threshold on the correlation coefficient of the noise evidenced in the Robinson–Jeffress experiment can be made to coincide with the dependence obtained in the noise-delay experiment if (a) the correlation coefficient is computed only at values for which the phase of the carrier of the correlation function is an integral multiple of π (i.e., only the envelope of the correlation function is considered) and (b) the transfer function of the critical-band filter is assumed to be a rectangle with a width of 100 cps. This result is consistent with the present model provided that one assumes that the auditory system is capable of inserting compensating delays up to, but not beyond, one-half of a period of the signal tone (1.0 msec at 500 cps).[21] In this case, the only affect of the carrier phase is to determine the interaural-signal phase after the compensating delay is inserted (the compensating delay affects the signal as well as the noise). As far as the interaural correlation of the noise is concerned, the only parameter that matters is the envelope value. Finally, it is also interesting to note that the value of 100 cps obtained by Langford and Jeffress for the critical bandwidth is identical to the value obtained by the writer in fitting the equalization-and-cancellation model to the Jeffress, Blodgett, and Deatherage data.

[19] For a comparison of these two experiments in terms of the correlation coefficient, see Ref. 6, Table I. For an analysis of the Jeffress–Blodgett–Deatherage data in terms of the equlization-and-cancellation model, see Ref. 3, pp. 1213–1217.

[20] L. A. Jeffress, private communication.

ACKNOWLEDGMENTS

The writer is indebted to the following people for useful discussions concerning the material presented in this note: P. R. Gray, J. L. Hall, II, L. A. Jeffress, and L. R. Rabiner.

This work was supported in part by the U. S. Army, U. S. Navy, and U. S. Air Force under contract DA 36–039–AMC–03200 (E); in part by the National Science Foundation (grant GP–2495); the National Institutes of Health, U. S. Department of Health, Education, and Welfare (grant MH–04737–04); and the National Aeronautics and Space Administration (grant NsG–496).

[21] If the auditory system is incapable of constructing delays greater than some maximum value τ_0 and $\tau_0<\pi/\omega_0$, then the correlation of the noise after the attempted compensation will depend on the carrier phase ϕ when ϕ is in the interval $\omega_0\tau_0<\phi\leq\pi$. If, on the other hand, the auditory system is capable of constructing all delays τ in the interval $0\leq\tau\leq\tau_0$ and $\tau_0>\pi/\omega_0$, then the correlation of the noise after the compensation will be independent not only of the carrier phase but also of the envelope, provided that the delay in the stimulus is confined to values less than or equal to τ_0.